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Navigation message designing with high accuracy for NAV

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Abstract Navigation message designing with high accuracy guarantee is the key to efficient navigation message distribution in the global navigation satellite system (GNSS). Developing high accuracy-aware navigation message designing algorithms is an important topic. This paper investigates the high-accuracy navigation message designing problem with the message structure unchanged. The contributions made in this paper include a heuristic that employs the concept of the estimated range deviation (ERD) to improve the existing well-known navigation message on L1 frequency (NAV) of global positioning system (GPS) for good accuracy service; a numerical analysis approximation method (NAAM) to evaluate the range error due to truncation (RET) of different navigation messages; and a basic positioning parameters designing algorithm in the limited space allocation. Based on the predicted ultra-rapid data from the ultra-rapid data from the international GPS service for geodynamic (IGU), ERDs are generated in real time for error correction. Simulations show that the algorithms developed in this paper are general and flexible, and thus are applicable to NAV improvement and other navigation message designs.

1. Introduction

The rapid development of the global navigation satellite system (GNSS) impels the navigation message structure to migrate from traditional fixed frames to flexible pages and navigation message contents to change from essential parameters to almost all-inclusive information. The current navigation message structure is now facing many challenges in efficiently deploying new emerged contents. A major impediment lies in its inability to deliver real-time orbit and clock corrections with high accuracy guarantee. To remove this limitation, a high accuracy-aware navigation message format, which distributes orbit/clock parameters and their corrections concurrently while satisfying the high-accuracy requirements, is proposed as one of the important techniques for efficient navigation message broadcasting. Thus, the navigation message with high accuracy plays a crucial role in the current GNSS evolution and is expected to be a key component of the BeiDou navigation system.

Although great progresses have been made in developing navigation message with high accuracy, they seemed incompatible with the existing practical navigation message on the L1 frequency (NAV) of global positioning system (GPS). This is partly due to the fact that the delivery of new contents demands reconstructing the message structure and redefining

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parameters. On the other hand, establishing a series of parameters' definition with the limited bit allocation constraint is still an untouched area. Therefore, navigation message designing with high accuracy guarantee for NAV becomes a significant research problem, which is the main focus of the research presented in this paper.

The navigation message designing problem was treated as a potential barrier first in the 1970s during the GPS designing process. Efforts were initially focused on the representation model selection and numerous requirement trade studies including refresh rate, time-to-first-fix, users' computational time, users' storage requirements,[1] et al. The published interface control documents (ICDs)[2–7] for GPS and the global navigation satellite system (GLONASS), which focused on the improvement of accuracy and the completeness of navigation message contents, have attracted great research interests. However, these research works[8–12] mainly focused on the comparison and analysis of navigation message structures and contents. The orbit and clock representations proposed by Van Dierendonck et al.[1] were widely adopted in GPS NAV and Galileo,[2–4] while GLONASS and civilized GPS ICDs[5–7] employed some different representations. Currently, GPS has two sets of ephemeris parameters, which have fifteen parameters in NAV and seventeen parameters in the civilized navigation message (CNAV), respectively. However, these works were carried out without taking the parameters' definition and the actual bit number in a navigation message into account.

In parallel, the relationship between the parameter definition and the allotted bit number is important in navigation message designing, when the model representation is fixed. The existing research aimed to seek a solution to obtain a series of parameters' definition with a range error due to truncation (RET) constraint.[1] The parameter definition included the number of bits and the scale factor (SF) of the least significant bit (LSB). Cui et al.[13] extended the SF designing method of ephemeris parameters from the traditional numerical analysis to an absolute value based one. With a definite RET constraint this method can obtain a series of ephemeris parameters whose definition is the same as that of NAV. However, the RET requirement and the evaluation strategy of the existent navigation message lack intensive study. The finding of an efficient parameters definition solution with a bit number constraint is still an untouched area. It is significant to find the relationship between the bits allocation and the RET.

The above referred researches are mainly focused on finding a set of parameters to describe an orbit with the representation model and RET constraints. Navigation message designing with more constraints, to the best of our knowledge, has not been studied yet. In this paper, the authors investigate the problem of accuracy improvement in the level of navigation message designing with three constraints, which are the RET, the bit allocation, and the representation model that decides the parameter sets. The paper aims to seek a trade-off solution among bit allocation, RET, and parameter definition with accuracy guarantee, which is applicable to BeiDou navigation message design.

2. Problem description

Navigation message contents can be represented by a series of parameters with fixed bits, effective range, and scale factor. Denote V as the set of n parameters in orbit and clock representation models. The fifteen ephemeris parameters and three clock parameters in NAV and the seventeen ephemeris parameters and three clock parameters in CNAV are two special cases of V. $\boldsymbol{S}_{1\times n}$, $\boldsymbol{E}_{1\times n}$, and $\boldsymbol{B}_{1\times n}$ are respectively the scale factor, the effective range, and the bit number order of the n parameters in V. b_i is obtained from s_i and e_i, according to whether the parameter has the sign bit ($+$ or $-$) occupying the most significant bit (MSB). Given a set of source parameters $p \in V$, orbit and clock parameters' designing is to obtain a set of destination parameters $D(\boldsymbol{S},\boldsymbol{E},\boldsymbol{B})$. T denotes the sum of the bit numbers and δr denotes the corresponding RET.

Problem 1: Given a source parameter set $p \in V$, the problem is to find a supplement to improve the accuracy (SIA).

Since the seventeen ephemeris parameters in CNAV excelled the ephemeris representation model in NAV in fitting error, now the existing NAV is facing an optimization problem. According to the ultra-rapid data from the international GPS service for geodynamic (IGU), some estimated range deviations (ERDs) are presented to improve the accuracy of the existing orbit and clock model.

Problem 2: Given a source parameter set $p \in V$ and the specific parameter definition $D(\boldsymbol{S},\boldsymbol{E},\boldsymbol{B})$, the problem is to compute the RET.

A numerical analytical method based on the Taylor series is utilized to estimate the RET. To any concrete orbit and clock design, the time-variant RET is calculated to evaluate the different navigation message content definitions.

Problem 3: Given a total bit number threshold T_0 and a source parameter set $p \in V$, the problem is to find a set of optimum destination parameters (ODPs) $D(\boldsymbol{S},\boldsymbol{E},\boldsymbol{B})$, while T does not exceed T_0.

Assuming the optimum parameters definition exists[1,13] when the RET is fixed, a mapping between δr and T can be found. δr breaks up the total bit number T into some intervals. Since the threshold may lie in a certain interval, the total bit number corresponds to a RET. An optimum set of parameters can be obtained through step-inputting the RET.

3. The proposed algorithm and analysis

3.1. A heuristic for SIA

The definition of the orbit and clock parameters in the GPS NAV and CNAV are listed in Table 1. The corresponding algorithms are included in Refs.[2–4].

Refs.[14–18] revealed that the position accuracy of an in-orbit satellite with NAV was ± 1.8 m. It has been proved that the positioning accuracy of the IGU[19] can reach 5 cm to GPS orbits, while 3 ns to GPS satellite clock. Then four ERDs (E_x, E_y, E_z, E_c), including the estimated range deviations in the X, Y, Z directions in the Earth-centered, Earth-fixed coordination (ECEF) and the clock, are presented to improve the accuracy. The ERDs are obtained from the IGU data. Each ERD is designed to be represented as a six-bit two's complement field with the sign bit occupying the MSB and LSB of 0.1 m for an effective range of ± 3.1 m. Every satellite only broadcasts its own ERDs, which update in every 30 s. A binary value of "100000" shall indicate that no valid ERD is present in that slot. The ERD bin structure which specifies the ERD index (ERDI) to ranges of ERD in meters is shown in Table 2.

Table 1 GPS orbit and clock parameters.

NAV	CNAV	Definition
Δn	Δn	Mean motion difference from computed value at reference time
Null	$\Delta \dot{n}$	Mean anomaly at reference time
$\sqrt{A}$	Null	Square root of the semi-major axis
Null	ΔA	Semi-major axis difference at reference time
Null	$\dot{A}$	Change rate in semi-major axis
e	e	Eccentricity
i_0	i_0	Inclination at reference time
Ω_0	Ω_0	Longitude of ascending node of orbit at reference time
ω	ω	Argument of perigee
M_0	M_0	Mean anomaly at reference time
t_{oe}	t_{oe}	Reference time for ephemeris
$\dot{i}$	$\dot{i}$	Rate of inclination angle
$\dot{\Omega}$	Null	Rate of right ascension
Null	$\Delta\dot{\Omega}$	Rate of right ascension difference
C_{is},C_{ic}	C_{is},C_{ic}	Amplitudes of harmonic correction terms for angle of inclination
C_{rs},C_{rc}	C_{rs},C_{rc}	Amplitudes of harmonic correction terms for orbit radius
C_{us},C_{uc}	C_{us},C_{uc}	Amplitudes of harmonic correction terms for argument of latitude
t_{oc}	t_{oc}	Reference time for clock
a_0	a_0	Clock bias correction
a_1	a_1	Clock drift correction
a_2	a_2	Clock drift rate correction

Table 2 ERDI and ranges of ERD.

ERDI	ERD (m)
−32	No ERD is available
$-31 \leqslant N \leqslant -1$	$0.1N < \mathrm{ERD} \leqslant 0.1\,(N+1)$
$0 \leqslant N \leqslant 31$	$0.1\,(N-1) < \mathrm{ERD} \leqslant 0.1N$

The transmitted ERDI is an integer value in the range of −31 to 31. The users shall utilize the value ERD = 0.1ERDI. They may apply the ERDs to the NAV message broadcast by each satellite using the following equation:

$$\begin{cases} x_{\mathrm{s}} = x_k - E_x, y_{\mathrm{s}} = y_k - E_y, z_{\mathrm{s}} = z_k - E_z \\ \Delta t_{\mathrm{s}}(t) = a_0 + a_1(t - t_{\mathrm{oc}}) + a_2(t - t_{\mathrm{oc}})^2 - E_{\mathrm{c}}/c \end{cases} \tag{1}$$

where c is the speed of light. x_k, y_k, z_k mean the satellite position calculated by the traditional ephemeris algorithm. x_{s}, y_{s}, z_{s} and $\Delta t_{\mathrm{s}}(t)$ mean the satellite position and the clock correction. Note that as shown above, the algebraic sense of an ERD value is opposite to those of the satellite vehicle (SV) position and clock correction terms. The ERD values are actually error estimates rather than differential corrections, so are subtracted rather than added.

3.2. A numerical analysis approximation method (NAAM) for RET

The scale factors of the LSBs were determined through a sensitivity analysis.[13] This analysis computed the sensitivity of the range error due to truncation of orbit and clock parameters. Since the required RET is unknown initially, it is of great necessity to learn the RET influenced by the bit allocation, and thus to evaluate the existing ICDs and to guide BeiDou navigation message designing. It is instructive to examine the general sensitivity of the range error to variations (parameters' truncation error at reference time) and test the relationship between RET and bit allocation. Except the ephemeris reference time, the fifteen broadcast NAV ephemeris parameters and the seventeen broadcast CNAV ephemeris parameters in Table 1 can be expressed as 15-element and 17-element vectors, respectively.

$$\begin{aligned} \boldsymbol{p}_{\mathrm{e}}^{1\times 15} &= [p_1, \quad p_2, \quad \ldots, \quad p_{15}] \\ &= [\Delta n, \sqrt{A}, e, i_0, \Omega_0, \omega, M_0, \dot{i}, \dot{\Omega}, C_{\mathrm{is}}, C_{\mathrm{ic}}, C_{\mathrm{rs}}, C_{\mathrm{rc}}, C_{\mathrm{us}}, C_{\mathrm{uc}}] \end{aligned}$$

$$\begin{aligned} \boldsymbol{p}_{\mathrm{e}}^{1\times 17} &= [p_1, \; p_2, \; \ldots, \; p_{17}] \\ &= [\Delta n, \Delta\dot{n}, \Delta A, \dot{A}, e, i_0, \Omega_0, \omega, M_0, \dot{i}, \Delta\dot{\Omega}, C_{\mathrm{is}}, C_{\mathrm{ic}}, C_{\mathrm{rs}}, C_{\mathrm{rc}}, C_{\mathrm{us}}, C_{\mathrm{uc}}] \end{aligned}$$

The vector of the three clock parameters is

$$\boldsymbol{p}_{\mathrm{c}}^{1\times 3} = [a_0, \quad a_1, \quad a_2]$$

To the NAV and CNAV ephemeris, the satellite position (x_{s}, y_{s}, z_{s}) at time t can be calculated as the Eq. (2) and Eq. (3), respectively:

$$x_{\mathrm{s}}(t) = F(\boldsymbol{p}_{\mathrm{e}}^{1\times 15}, t_{\mathrm{oe}}, t) = F(p_1, p_2, \ldots, p_{15}, t_{\mathrm{oe}}, t) \tag{2a}$$
$$y_{\mathrm{s}}(t) = G(\boldsymbol{p}_{\mathrm{e}}^{1\times 15}, t_{\mathrm{oe}}, t) = G(p_1, p_2, \ldots, p_{15}, t_{\mathrm{oe}}, t) \tag{2b}$$
$$z_{\mathrm{s}}(t) = H(\boldsymbol{p}_{\mathrm{e}}^{1\times 15}, t_{\mathrm{oe}}, t) = H(p_1, p_2, \ldots, p_{15}, t_{\mathrm{oe}}, t) \tag{2c}$$

$$x_{\mathrm{s}}(t) = I(\boldsymbol{p}_{\mathrm{e}}^{1\times 17}, t_{\mathrm{oe}}, t) = I(p_1, p_2, \ldots, p_{17}, t_{\mathrm{oe}}, t) \tag{3a}$$
$$y_{\mathrm{s}}(t) = J(\boldsymbol{p}_{\mathrm{e}}^{1\times 17}, t_{\mathrm{oe}}, t) = J(p_1, p_2, \ldots, p_{17}, t_{\mathrm{oe}}, t) \tag{3b}$$
$$z_{\mathrm{s}}(t) = K(\boldsymbol{p}_{\mathrm{e}}^{1\times 17}, t_{\mathrm{oe}}, t) = K(p_1, p_2, \ldots, p_{17}, t_{\mathrm{oe}}, t) \tag{3c}$$

where $F(\cdot)$, $G(\cdot)$, and $H(\cdot)$ are nonlinear functions defined by the satellite position algorithms in GPS ICD-200F.[2] $I(\cdot)$, $J(\cdot)$, and $K(\cdot)$ are nonlinear functions defined by the satellite position algorithms in GPS ICD-705 and ICD-800.[3,4]

The satellite clock correction can be computed as

$$\Delta t_{\mathrm{s}}(t) = L(\boldsymbol{p}_{\mathrm{c}}^{1\times 3}, t_{\mathrm{oc}}, t) \tag{4}$$

where L is a nonlinear function defined by the satellite clock algorithms.[2–4]

The instantaneous range from the satellite to the Earth can be written as

$$r = \sqrt{x_s^2 + y_s^2 + z_s^2} + c\Delta t_s \tag{5}$$

Defining the instantaneous truncation as a vector: $\delta \boldsymbol{p} = [\delta \boldsymbol{p}_e, \delta \boldsymbol{p}_c]$, $\delta \boldsymbol{p} = \boldsymbol{p}_o - \boldsymbol{p}_t$, in which $\boldsymbol{p}_o$ and $\boldsymbol{p}_t$ are vectors of orbit and clock parameters without and with truncation, respectively. To every parameter, the truncation error obeys a uniform distribution between $-s_i$ and s_i. s_i means the scale factor of the parameter. Then the general sensitivity of the range error to the parameters' variations due to truncation is

$$\delta r = \left(\frac{\partial r}{\partial x_s} \frac{\partial x_s}{\partial \boldsymbol{p}} + \frac{\partial r}{\partial y_s} \frac{\partial y_s}{\partial \boldsymbol{p}} + \frac{\partial r}{\partial z_s} \frac{\partial z_s}{\partial \boldsymbol{p}} + \frac{\partial r}{\partial \Delta t_s} \frac{\partial \Delta t_s}{\partial \boldsymbol{p}} \right) \Bigg|_{\boldsymbol{p}_o} \delta \boldsymbol{p}^{\mathrm{T}} \tag{6}$$

Now consider the sensitivity to the 15-element parameter variations:[20]

$$\begin{bmatrix} \delta x_s(t) \\ \delta y_s(t) \\ \delta z_s(t) \end{bmatrix} = \begin{bmatrix} \dfrac{\partial F(\boldsymbol{p}_e^{1\times15}, t_{oe}, t)}{\partial \boldsymbol{p}_e^{1\times15}} \\ \dfrac{\partial G(\boldsymbol{p}_e^{1\times15}, t_{oe}, t)}{\partial \boldsymbol{p}_e^{1\times15}} \\ \dfrac{\partial H(\boldsymbol{p}_e^{1\times15}, t_{oe}, t)}{\partial \boldsymbol{p}_e^{1\times15}} \end{bmatrix} (\delta \boldsymbol{p}_e^{1\times15})^{\mathrm{T}} \tag{7}$$

where $\delta x_s, \delta y_s, \delta z_s$ are the deviations in the direction X, Y, Z directions due to the ephemeris parameters' truncation. Since the algorithm of the 17-element ephemeris parameters is different from that of the 15-element ones, the sensitivity to the 17-element parameter variations is

$$\begin{bmatrix} \delta x_s(t) \\ \delta y_s(t) \\ \delta z_s(t) \end{bmatrix} = \begin{bmatrix} \dfrac{\partial I(\boldsymbol{p}_e^{1\times17}, t_{oe}, t)}{\partial \boldsymbol{p}_e^{1\times17}} \\ \dfrac{\partial J(\boldsymbol{p}_e^{1\times17}, t_{oe}, t)}{\partial \boldsymbol{p}_e^{1\times17}} \\ \dfrac{\partial K(\boldsymbol{p}_e^{1\times17}, t_{oe}, t)}{\partial \boldsymbol{p}_e^{1\times17}} \end{bmatrix} (\delta \boldsymbol{p}_e^{1\times17})^{\mathrm{T}} \tag{8}$$

The sensitivity to the clock 3-element parameter variations is

$$\delta(c\Delta t) = \frac{\partial L(\boldsymbol{p}_c^{1\times3}, t_{oc}, t)}{\partial \boldsymbol{p}_c^{1\times3}} (\delta \boldsymbol{p}_c^{1\times3})^{\mathrm{T}} \tag{9}$$

where $\delta(c\Delta t)$ is the range deviation due to the clock parameters' truncation. $\delta \boldsymbol{p}_e^{1\times15}$ is the NAV ephemeris parameters' truncation vector, while $\delta \boldsymbol{p}_e^{1\times17}$ is the CNAV ephemeris parameters' truncation vector. Then the following equation can be obtained:

$$\delta \boldsymbol{r}(t) = \begin{bmatrix} A_e^{3\times15}(\boldsymbol{p}_e^{1\times15}, t_{oe}, t) & \boldsymbol{0}^{3\times3} \\ \boldsymbol{0}^{1\times15} & A_c^{1\times3}(\boldsymbol{p}_c^{1\times3}, t_{oc}, t) \end{bmatrix} \begin{bmatrix} (\delta \boldsymbol{p}_e^{1\times15})^{\mathrm{T}} \\ (\delta \boldsymbol{p}_c^{1\times3})^{\mathrm{T}} \end{bmatrix} \tag{10}$$

where $\delta \boldsymbol{r}(t)$ is the deviations vector due to the ephemeris and clock parameters' truncation variations.

$$\delta \boldsymbol{r}(t) = [\delta x_s(t), \quad \delta y_s(t), \quad \delta z_s(t), \quad \delta(c\Delta t)]^{\mathrm{T}} \tag{11}$$

The relationship between $\delta \boldsymbol{r}(t)$ and the scalar $\delta r(t)$ is

$$\delta r(t) = \sqrt{(\delta x_s(t))^2 + (\delta y_s(t))^2 + (\delta z_s(t))^2} + \delta(c\Delta t_s) \tag{12}$$

The seventeen-element ephemeris can be written as:

$$\delta r(t) = \begin{bmatrix} A_e^{3\times17}(\boldsymbol{p}_e^{1\times17}, t_{oe}, t) & \boldsymbol{0}^{3\times3} \\ \boldsymbol{0}^{1\times17} & A_c^{1\times3}(\boldsymbol{p}_c^{1\times3}, t_{oc}, t) \end{bmatrix} \begin{bmatrix} (\delta \boldsymbol{p}_e^{1\times17})^{\mathrm{T}} \\ (\delta \boldsymbol{p}_c^{1\times3})^{\mathrm{T}} \end{bmatrix} \tag{13}$$

The expressions of the sensitivity matrix are as follows:

$$A_e^{3\times15} = \begin{bmatrix} \dfrac{\partial r}{\partial F} \dfrac{\partial F(\boldsymbol{p}_e^{1\times15}, t_{oe}, t)}{\partial \boldsymbol{p}_e^{1\times15}} \\ \dfrac{\partial r}{\partial G} \dfrac{\partial G(\boldsymbol{p}_e^{1\times15}, t_{oe}, t)}{\partial \boldsymbol{p}_e^{1\times15}} \\ \dfrac{\partial r}{\partial H} \dfrac{\partial H(\boldsymbol{p}_e^{1\times15}, t_{oe}, t)}{\partial \boldsymbol{p}_e^{1\times15}} \end{bmatrix} \tag{14}$$

$$A_e^{3\times17} = \begin{bmatrix} \dfrac{\partial r}{\partial I} \dfrac{\partial I(\boldsymbol{p}_e^{1\times17}, t_{oe}, t)}{\partial \boldsymbol{p}_e^{1\times17}} \\ \dfrac{\partial r}{\partial J} \dfrac{\partial J(\boldsymbol{p}_e^{1\times17}, t_{oe}, t)}{\partial \boldsymbol{p}_e^{1\times17}} \\ \dfrac{\partial r}{\partial K} \dfrac{\partial K(\boldsymbol{p}_e^{1\times17}, t_{oe}, t)}{\partial \boldsymbol{p}_e^{1\times17}} \end{bmatrix} \tag{15}$$

$$A_c^{1\times3} = \frac{\partial r}{\partial \Delta t} \frac{\partial L(\boldsymbol{p}_c^{1\times3}, t_{oc}, t)}{\partial \boldsymbol{p}_c^{1\times3}} \tag{16}$$

where $A_e^{3\times15}$ is the 3×15 sensitivity matrix which can be computed either analytically or numerically by partial differentiation of F, G, and H; $A_e^{3\times17}$ is the 3×17 sensitivity matrix which can be computed either analytically or numerically by partial differentiation of I, J, and K. In a more compact form, the sensitivity of the range error to parameters' truncation variations can be written as

$$\delta \boldsymbol{r}(t) = \boldsymbol{A}\delta \boldsymbol{p} \tag{17}$$

The equation is a linearized expression directly relating the range error and the parameters' truncation variations. Each term in the sensitivity matrix is a function of the reference time and the broadcast parameters. Thus the existing ICDs can be evaluated.

3.3. Parameter designing algorithm for ODP

To design the basic positioning parameters within a limited bit allocation, a mapping between the range error and total bits needs to be obtained first. The component of the instantaneous truncation vector is $\delta p_i = p_i - \lceil \frac{p_i}{s_i} \rceil s_i$, where $\lceil\ \rceil$ refers to the truncation to an integer.

As described above, the user's range to the satellite can be expanded in a Taylor's series at any given time:

$$r = r^* + \frac{\partial r}{\partial \boldsymbol{p}}\bigg|_{\boldsymbol{p}_o} \delta \boldsymbol{p} + o(\delta \boldsymbol{p}^2) \tag{18}$$

That is

$$\delta r = r - r^* = \frac{\partial r}{\partial \boldsymbol{p}}\bigg|_{\boldsymbol{p}_o} \delta \boldsymbol{p} + o(\delta \boldsymbol{p}^2) \approx \frac{\partial r}{\partial \boldsymbol{p}}\bigg|_{\boldsymbol{p}_o} \delta \boldsymbol{p} \tag{19}$$

It is assumed that the contribution to the range error is uniform over the components, that is

$$\left| \frac{\partial r}{\partial \boldsymbol{p}}\bigg|_{\boldsymbol{p}_o} \delta \boldsymbol{p}_i \right| < \frac{R}{n} \quad \text{or} \quad \delta \boldsymbol{p}_i < \frac{R}{n} \Bigg/ \left| \frac{\partial r}{\partial \boldsymbol{p}}\bigg|_{\boldsymbol{p}_o} \right| \tag{20}$$

where R indicates the requirement of RET. As to GPS NAV, the required RET of orbit parameters is 0.3 m. The inequality is used to determine the most significant bit of δp_i. The LSB scale factor of the components of p_i is chosen to be one bit larger. Fractions of bits are dropped. Based on this method, an optimum combination of ephemeris definition can be obtained with a determined range error caused by truncation. The parameters' designing algorithm is as follows:

(1) Input the ephemeris representations V and the total bit number limitation T_0.
(2) Compute the parameters $\boldsymbol{p}$ and the sensitivity matrix $\boldsymbol{A}$ for every satellite in the constellation, when $t = 7199$ s.
(3) Choose $r = 0.1i$, $i \in [1,10]$, simulate the maximum T to all satellites as the total bits T_i. Obtain the mapping between the RET r and the total bits T_i.
(4) If $T_0 = T_i$, set $r = r_i$, $T = T_i$. Output T and $D(\boldsymbol{S},\boldsymbol{E},\boldsymbol{B})$.
(5) If $T_0 \in (T_i, T_{i+1})$, set $r = r_i + 0.01j$, $j \in [1,10]$. Obtain the mapping between the RET r and the total bit T_j.
(6) Find out the interval T_0 locate in, such as $T_j \leqslant T_0 < T_{j+1}$. Then set $r = r_j$, $T = T_j$. Output T and $D(\boldsymbol{S},\boldsymbol{E},\boldsymbol{B})$.

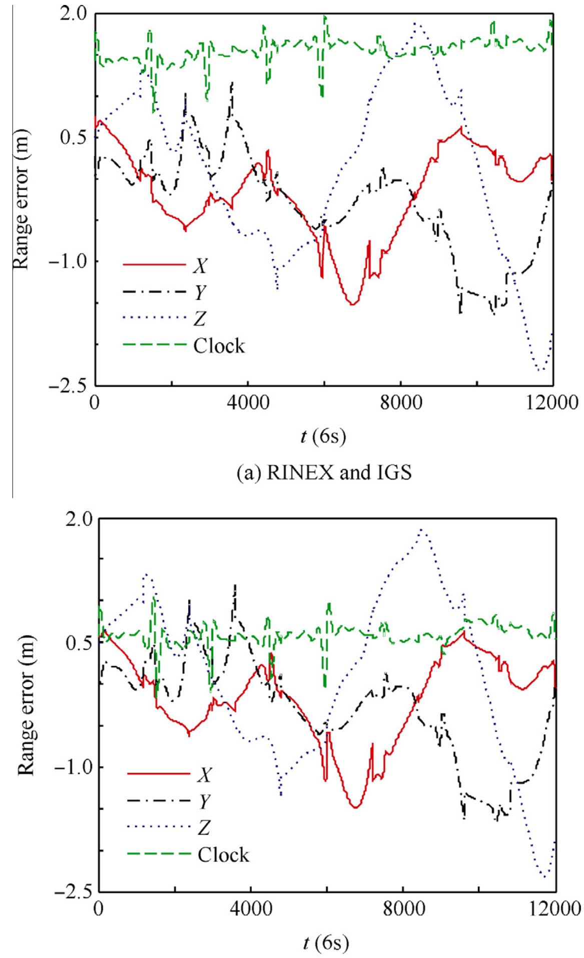
4. Simulation results

To validate the proposed methods and algorithm, the authors implemented experiments with the broadcast receiver independent exchange format (RINEX) data, the IGU and the final orbit data from the international GPS service (IGS) for geodynamic data. The simulations involve three parts, that is, validating the supplements to improve the accuracy, evaluating the RETs of the GPS NAV and CNAV, and designing a feasible series of parameters for NAV to verify the quality of the parameters designing algorithm and the proposed ERDs.

4.1. SIA validation

A great deal of research[15–18] has been done to test the accuracy of the current GPS broadcast navigation message. In the first set of experiments, orbit errors and clock errors are computed by comparing the broadcast ephemeris/clock data with the precise IGS data. Since the IGU data are released one day ahead, the ERDs are obtained from the differences between the RINEX and IGU data, followed by the generation of ERDs on the X, Y, Z, and clock directions every 6 s. To verify the quality of the supplement parameter ERD, the RINEX data with ERDs compensation are compared with the IGS data. The IGS and IGU data are interpolated with a trigonometric function to obtain the six seconds' interval values, while 6 s is the lasting time of one page for NAV. The tested time ranges 22 h, and the results are presented in Figs. 1–3.

Fig. 1(a) gives the range error of the broadcast navigation message. As it can be seen, the orbit errors range from −2.5 m to 2.0 m, while the clock error ranges from 0.8 m to 2.0 m. Meanwhile, Fig. 1(a) shows that the trends of the orbit errors due to ephemeris are very smooth, while the trend of the range error due to clock usually jumps substantially. The clock error has the worst performance compared to the three directions. Fig. 1 shows the orbit errors between RINEX and IGU are of the same trends as that between RINEX and IGS. This means the errors can be considerably reduced by the IGU data. Fig. 2 plots the 6 s updated ERDs' indexes which are generated by comparing the RINEX and IGU data.



(a) RINEX and IGS

(b) RINEX and IGU

Fig. 1 Differences among RINEX and the international GPS orbit data.

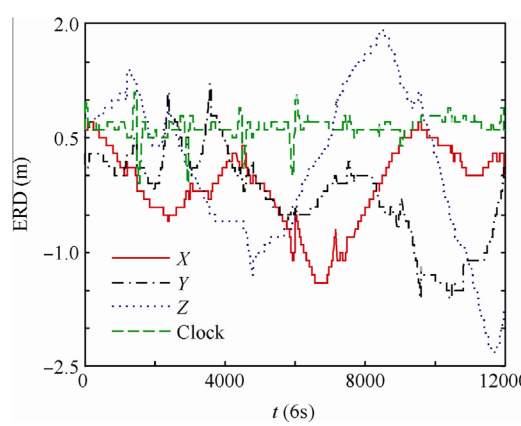


Fig. 2 Updated ERDs' indexes generated by comparing RINEX and IGU.

Heng, et al.[16–18] concluded that the signal-in-space performance was dominated by the clock performance and the clock performance affected the ephemeris performance. Based on the last seven years' data comparison, the long-term performance for all satellites revealed that the worst cases of orbit and clock

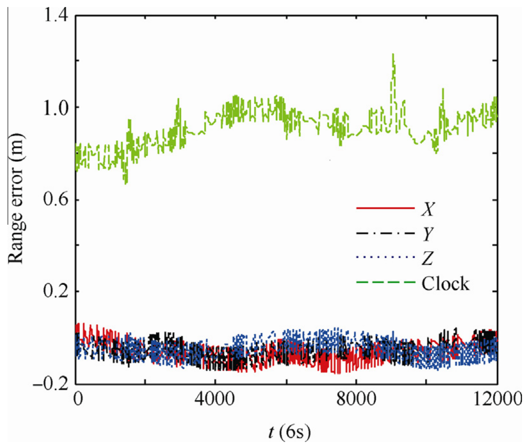


Fig. 3 Error correction effect.

errors were less than 3.0 m. Since the bit allocation is limited and the orbit errors will be projected to the direction from a user to the satellite, the supplement ERDs are the error corrections introduced to make up the orbit and clock errors. ERDs are designed to be represented as a six-bit two's complement field with the sign bit occupying the MSB and the LSB of 0.1 m. Fig. 3 gives the range errors while ERDs are applied. As expected, the orbit errors decrease to 0.1 m and the clock error is reduced to 1.2 m followed by a lot of sudden changes disappearing. That is to say, the ERDs can considerably decrease and smooth the orbit and clock errors. This obeys the previous theoretical expectation.

4.2. NAAM application

In the second set of experiments, the RET variations of three impact factors including ephemeris elements and clock elements as well as scale factors, versus the navigation message system time were tested respectively to evaluate the RET of the existing NAV and CNAV navigation message.

Fig. 4 plots the range error due to the truncated parameters in NAV and CNAV. The tested time lasts one month, and the sampling interval is chosen to be 2 h. It can be seen from Fig. 4, the RET mainly comes from the truncated clock parameters in NAV. As for CNAV, the RET mainly comes from the truncated ephemeris parameters. CNAV is superior to NAV in RET, especially in the clock parameters' definition.

Meanwhile, Fig. 5 is given to describe the relationship between the RET, the different ephemeris/clock elements, and the total bits. Fig. 5 shows the performance of total bits is very steady along with the increasing range error. The 17-element ephemeris occupies more bits than the 15-element ephemeris. Moreover, it is interesting to note that the gradually increasing range error changing from 0.5 m to 2.5 m has little impact on the total bits for each parameter set, while the range error ranging from 0 m to 0.5 m influences the total bits greatly. Even so, the results still accord with the previous theoretical derivations: more bits of parameters can bring less RET. Thus, a conclusion is drawn that the NAAM can evaluate the existing navigation message definition efficiently and the mapping in Fig. 5 can guide the designing of the basic positioning parameters in the navigation message.

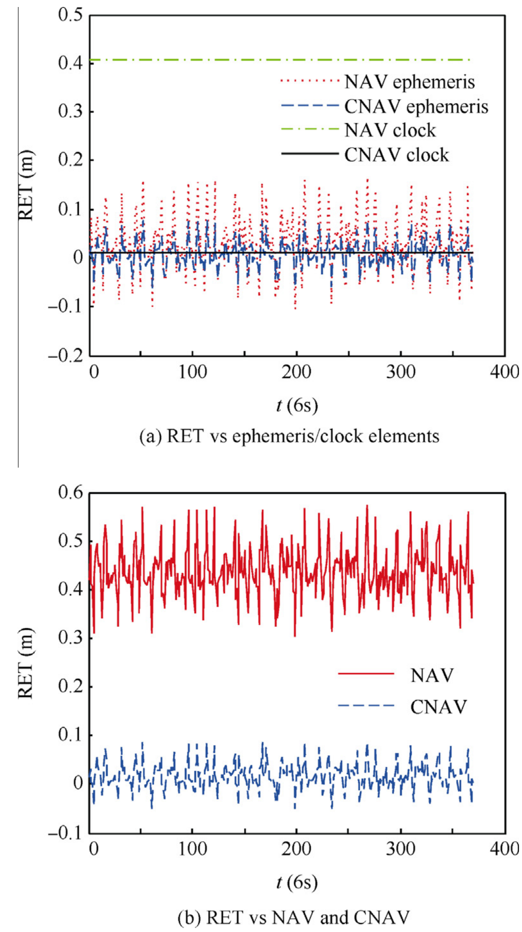


Fig. 4 RET evaluation between NAV and CNAV.

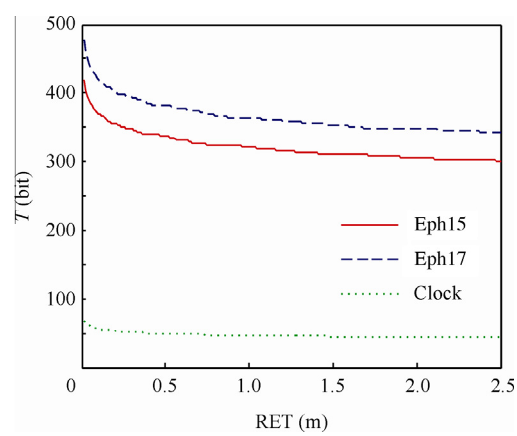


Fig. 5 Total bits and RET.

4.3. ODP example

The NAV data message has five subframes. The first subframe contains the satellite's clock correction parameters and the group delay parameters. The second and third subframes contain the satellite's ephemeris. Currently, there are not enough bits reserved to hold the additional ERD parameters. Since compression cannot be done in some pages in the fourth and fifth subframes, it is impossible to broadcast the ERD in every 6 s. There are 16 bits reserved in the first subframe to contain the ERD on clock. It is of great necessity to compress the existing navigation message contents in the second and the third subframes to broadcast the orbit ERDs every 30 s. Since the reference time t_{oe} can be condensed to 11 bits with a scale factor of 300 s, the ODP problem comes down to a problem with $T_0 = 329$ and the source parameter set $\boldsymbol{p}_e^{1\times15}$. The mapping in Fig. 5 can be applied to an off-line computation manner in guiding the navigation message design. A brief introduction of the orbit and clock parameters as well as the ERDs design results in the NAV navigation message as given in Table 3 with the parameter designing algorithm. Since the clock parameters in the first subframe and the supplementary ERD on clock can keep the clock error less than 1.5 m, the RET for clock parameters is less than 1.5 m. Then the clock parameters' definition is kept unchanged in the improvement process of the NAV message.

Thus only the ephemeris parameters' definition will change. Fig. 6 shows the accuracies of the new defined ephemeris parameters with ERDs on orbit and the IGS data. Compared with Fig. 3, it is apparent to see that the range error increases to 0.3 m with the new definition of ephemeris parameters. This demonstrates that ERDs can decrease the range error by a large margin, while the parameters' definition is the bottleneck. Nevertheless, the new definition combined with the ERDs can greatly improve the accuracy to a decimeter level. In conclusion, ODP finds a parameters' definition solution quickly and guarantees the RET quality of the parameters. To sum up, the experimental results validate that the previous theoretical derivations are feasible in reality.

Table 3 Parameter definition.

Term	No. of bits	SF	Effective range
C_{rc},C_{rs} (m)	15*	2^{-4}	± 1024
Δn (π/s)	15*	2^{-42}	$\pm 3.725 \times 10^{-9}$
M_0 (π)	30*	2^{-29}	± 1
C_{uc},C_{us} (rad)	17*	2^{-30}	$\pm 6.103 \times 10^{-5}$
e	29	2^{-30}	0.03
$\sqrt{A}(\sqrt{\text{m}})$	32	2^{-19}	8192
C_{ic},C_{is} (rad)	16*	2^{-29}	$\pm 6.103 \times 10^{-5}$
Ω_0 (π)	30*	2^{-29}	± 1
i_0 (π)	30*	2^{-29}	± 1
$\dot{i}_0$ (π/s)	13*	2^{-42}	$\pm 9.31 \times 10^{-10}$
Ω (π)	31*	2^{-30}	± 1
$\dot{\Omega}$ (π/s)	23*	2^{-42}	$\pm 9.536 \times 10^{-7}$
E_x (m)	6*	0.1	± 3.1
E_y (m)	6*	0.1	± 3.1
E_z (m)	6*	0.1	± 3.1
E_c (m)	6*	0.1	± 3.1
t_{oe} (s)	11	300	604784

* Parameters so indicated shall be two's complement, with the sign bit (+ or −) occupying the MSB.

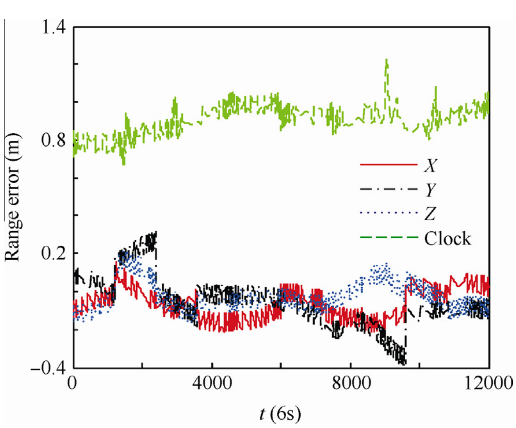


Fig. 6 Differences between new ephemeris parameters with ERDs and IGS.

5. Conclusions

(1) By aggregating the IGU and IGS data, ERDs on the X, Y, Z, and clock directions are heuristically generated to improve the accuracy of NAV.
(2) A numerical analysis approximation method (NAAM) is proposed to give a mapping among RET, bit allocation, and parameter sets.
(3) The NAAM is utilized to evaluate the existing navigation message and guide its designing. CNAV outperforms NAV in RET. It is preferable to increase the RET and the bit allocation for clock parameters.
(4) With the newly designed ephemeris parameters and the supplement ERDs hybridized in the real NAV message, the orbit error can be reduced to less than 0.4 m and the clock error can be held within 1.3 m without big jumps.

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